

# Adapting to Performance Variations in Multi-Robot Coverage

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**Abstract**—This paper proposes a new approach for a group of robots carrying out a collaborative task to adapt on-line to actuation performance variations among the robots. We consider the problem of multi-robot coverage, where a group of robots has to spread out to cover the environment. We suppose that some robots have poor actuation performance (e.g. weak motors, friction losses in the gear train, wheel slip, etc.) and some have strong actuation performance (powerful motors, little friction, favorable terrain, etc.). The robots do not know before hand the relative strengths of their actuation compared to the others in the team. The algorithm in this paper learns the relative actuation performance variations among the robots on-line, in a distributed fashion, and automatically compensates by giving the weak robots a small portion of the environment, and giving the strong robots a larger portion. Using a Lyapunov-type proof, we prove that the robots converge to locally optimal positions for coverage. The algorithm is demonstrated in both Matlab simulations and experiments using Pololu m3pi robots.

## I. INTRODUCTION

For multi-robot systems to perform robustly and practically in real-world settings, it is necessary for the robots to adapt to individual deficiencies and performance variations among the team. Our work considers a team of robots carrying out a coverage control task, in which the robots must spread out over the environment. Once deployed, the robots may need to perform sub-tasks that require movement around the environment. The team is heterogeneous in that some robots have better actuation performance than others. The difference may be due to different hardware components in the robots, differences in aging and degradation over time, or differences in the underlying terrain on which the robots are moving. In any case, we assume that some robots move faster and more accurately towards their goal than others. Furthermore, the robots do not know their relative performance with respect to the group, as would be the case in a real world setting. We propose an algorithm that incorporates online learning and Voronoi based coverage control. Using this algorithm, the robots learn a “performance weighting” indicating their own performance relative to the others in the group, using only local communication, and knowledge of their own actuation errors measured online. The effect of a low “performance weighting” is to shrink the

size of the robot’s dedicated coverage area, while robots with a high “performance weighting” take charge of covering a larger portion of the environment.

This ability to adapt online to actuation performance variations is important in several examples. First, consider the problem of illegal fishing, which accounts for approximately 1 in 5 fish caught in the wild [1]. Here, robots could use a Voronoi-based coverage control framework to distribute themselves across the environment. As boats enter their regions of interest, these robots may need to move closer or track patterns of the boats to identify illegal activity. Errors in actuation can compromise how quickly a robot could track a boat in its region. Another example is a group of robots working in a warehouse. In this scenario, robots are assigned regions of the warehouse and must retrieve items within their region as necessary. Actuation errors could limit the robot’s speed and precision and impair the group’s efficiency. Our algorithm accounts for this actuation quality and adapts the performance weights accordingly so that items are retrieved efficiently despite robot performance variations. Consider another situation in which robots are covering an environment with highly variable terrain (e.g. paved in some places, forested in other, sandy in others). In this case the wear and tear on the vehicles as well as the terrain itself will affect actuation across the terrain. These variations can be accounted for with our algorithm, so that the team maintains efficient coverage despite the heterogeneous terrain. Indeed, many coverage scenarios with real world performance variations can benefit from our algorithm.

Our algorithm builds upon the Voronoi based coverage strategy first proposed by Cortés et al. [2], [3]. This algorithm, often referred to as the move-to-centroid controller, all robots continuously drive toward the centroids of their Voronoi cells. This builds upon previous work in optimally locating retail facilities [4], as well as applications in data compression (i.e. “vector quantization”) [4]. Other extensions of Voronoi coverage control have used the weighted Voronoi diagram, also called the power diagram, where the weightings account for heterogeneity among robots. Using the Power Diagram, Pavone et. al showed that different cell weights allowed for agents to take on varying sensing responsibility [5]. Within a heterogeneous group of robots, the weighting can be used to correspond to the sensing radius of the robot [6]. Another approach used weighted Voronoi cells as an energy-efficiency metric, allowing the group to compensate for low-energy robots [7]. By using the Voronoi weights to quantify sensor health, Marier et. al assigned low-performing robots smaller areas of coverage ([8], [9]).

Within this variety of existing research on weighted

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Voronoi cells, most assume the correct weightings are known *a priori*. In contrast, our work seeks to *learn* performance weightings *online* using only information about the robot’s actuation, and data from its neighbors. This paper builds on the authors’ previous work, which incorporated sensing performance into an adaptive trust weighting [10]. Here, we consider actuation error instead of sensing errors. Actuator performance is significantly more challenging to learn than sensing performance because it requires learning a gain matrix using all past history of actuation errors, leading to a two part adaptation law for the performance weights. In contrast, our previous work learns a scalar value (indicating sensor performance) with a one part adaptation law. In this work we derive a two part adaptation law for each robot to (i) learn an unknown control gain matrix that quantifies actuation errors, and (ii) use this to adapt a performance weight to penalize or reward the robots based on its learned gain matrix. This adaptation occurs simultaneously while the robots carry out a Voronoi-based coverage control algorithm. Using a Lyapunov style proof, we show that the positions of the robots converge to a locally optimal coverage configuration, while the weightings converge to a set of values defined by the actuation error. We demonstrate the performance of the algorithm in Matlab simulations, and in hardware experiments with Pololu m3pi robots.

## II. PROBLEM SET-UP

Consider a set of  $n$  robots in a bounded, convex environment  $Q \subset \mathbb{R}^2$ . Points in  $Q$  are denoted  $q$ , and the position of the  $i$ -th agent is  $p_i \in Q$ . Prior coverage control algorithms use the standard Voronoi partition of the environment. Let  $\{V_1, \dots, V_n\}$  be the Voronoi partition of  $Q$ , with each cell defined as

$$V_i = \{q \in Q \mid \|q - p_i\| \leq \|q - p_j\|, \quad \forall j \neq i\}.$$

For our work, we use the weighted Voronoi partition, also known as the Power Diagram, with each weighting  $w_i$  serving as the performance weighting for robot  $i$ . Let  $\{W_1, \dots, W_n\}$  be the weighted Voronoi partition of  $Q$ , with each cell defined as

$$W_i = \{q \in Q \mid \|q - p_i\|^2 - w_i \leq \|q - p_j\|^2 - w_j, \quad \forall j \neq i\}. \quad (1)$$

For our bounded region  $Q$ , we also define an integrable function  $\phi : Q \rightarrow \mathbb{R}_{>0}$  to represent the areas of importance in the environment. Areas with large values of  $\phi(q)$  are more important than those with small values, and all the robots have knowledge of this function. When the robots do not know this function, techniques have been developed to learn it online from sensor data [11], [12].

### A. Locational Optimization

Before introducing our problem formulation, we will state some basic nomenclature and results from Voronoi based coverage control. A complete discussion can be found in [3], [5]. Using our definition of the weighted Voronoi cell

given in (1), we can formulate a cost function for coverage over some area  $Q$  as

$$\mathcal{W}(p_1, \dots, p_n, w_1, \dots, w_n) = \sum_{i=1}^n \int_{W_i} \frac{1}{2} (\|q - p_i\|^2 - w_i) \phi(q) dq, \quad (2)$$

where  $W_i$  is the robot’s weighted Voronoi cell, and  $w_i$  is the robot’s individual performance weighting. Intuitively, a low value of  $\mathcal{W}$  indicates a good configuration of robots for coverage of the environment. We can also define the quantities  $M_{W_i}$  and  $C_{W_i}$ , analogous to the physical masses and centroids of the Voronoi cells, calculated as

$$M_{W_i} = \int_{W_i} \phi(q) dq \quad \text{and} \quad C_{W_i} = \frac{1}{M_{W_i}} \int_{W_i} q \phi(q) dq.$$

Although there is a complex dependency between the robot position and the geometry of the Voronoi cells, a surprising result from locational optimization [4] is that the critical points of  $\mathcal{W}$  correspond to the configurations in which all the robots are located at the centroid of their Voronoi cell, or  $p_i = C_{W_i}$ , for all  $i$ . Cortés introduced a gradient-based controller that is guaranteed to drive the robots to the critical points corresponding to local minimum [3] and which has been generalized to the weighted Voronoi cell ([5], [8]). We restrict ourselves to only considering local minima of  $\mathcal{W}$  since global optimization of (2) is known to be difficult (NP-hard). Thus, when we refer to optimal coverage configurations, we mean locally optimal configurations. Variations on the control law which attempt to find global minima through exploration are discussed by Salapaka et al. [13] and Schwager et al. [14].

### B. Robot and Actuator Model

In this section, we describe our model for the dynamics of the robots and the quality of the actuation. First, we assume that the robots have integrator dynamics, where

$$\dot{p}_i = u_i + \Delta_i. \quad (3)$$

Here,  $u_i$  is the control input to the robot, and  $\Delta_i$  is an actuation uncertainty. We can equivalently assume there are low-level controllers in place to cancel existing dynamics and enforce (3). We also assume that the robots will be able to communicate with their neighbors and share information about their actuation error. We define the communication network as an undirected graph in which two robots share an edge of the graph if they share Voronoi cell boundaries, also known as the Delaunay graph. We can then write the set of neighbors for any robot  $i$  as  $\mathcal{N}_i := \{j \mid V_i \cap V_j \neq \emptyset\}$ . Additionally, robots are able to compute their own weighted Voronoi cells, as defined by (1), which is a common assumption in the literature ([3], [8], [13]).

## III. DECENTRALIZED CONTROL

The main goals of our work are to 1) drive the robots to an optimal coverage configuration in the environment and 2) adjust weightings to account for variations in performance

in the positional controller. To accomplish these goals, we propose one control law to change the positions of the robots and one adaptation law to change the weightings of the robots. We will then prove that both of these control laws will drive the robots to converge asymptotically to a stable equilibrium configuration corresponding to a local minimum of the cost function.

With respect to the position controller we will use

$$u_i = K_p(C_{W_i} - p_i), \quad K_p = \begin{bmatrix} k_p & 0 \\ 0 & k_p \end{bmatrix},$$

where  $k_p$  is a positive proportional gain constant. We will assume that the actuation errors can be quantified as

$$\Delta_i = K_{\Delta_i} K_p^{-1} u_i,$$

where  $K_{\Delta_i}$  is a matrix. This yields the overall closed loop equation

$$\dot{p}_i = u_i + \Delta_i = K_i(C_{W_i} - p_i), \quad (4)$$

where  $C_{W_i}$  is the centroid of the weighted Voronoi cell and

$$K_i = K_p + K_{\Delta_i}.$$

This controller is a modification of the move-to-centroid control law, first proposed by Cortés ([3]) and extended and modified in ([15],[8], [11]). While the original control law used the unweighted Voronoi cell centroid,  $C_{V_i}$ , it does not impact the performance of the controller to use the weighted Voronoi centroid,  $C_{W_i}$  ([8], [6]). In the unlikely case of an empty Voronoi cell, we evaluate (4) using the integral form of the gradient descent based controller, letting  $u_i = 0$ .

For the purposes of this paper, we can consider the  $K_{\Delta_i}$  matrix as one way to capture the imperfections in the movement of the robot. This paper considers the errors such that  $K_i$  is still a positive definite matrix. In practice, we may not know the exact value of  $K_{\Delta_i}$ , so we will introduce an estimator,  $\hat{K}_i$ , derived from known quantities. From the estimate  $\hat{K}_i$ , the robot can then adjust for positional errors with the following adaptation law

$$\dot{w}_i = \frac{-k_w}{M_{W_i}} \sum_{j \in \mathcal{N}_i} \left( (w_i - f(\hat{K}_i)) - (w_j - f(\hat{K}_j)) \right) \quad (5)$$

where  $k_w$  is a positive proportional gain constant and  $f(\hat{K}_i)$  is some function of the properties of  $\hat{K}_i$ . Note that the choice of  $f(\hat{K}_i)$  is subjective, based on the desired performance metrics of the system. The adaptation controller is inspired by consensus algorithms ([16],[17], [18]).

#### A. Estimating $K_i$

In practice, it is unlikely that the robots have direct knowledge of the actuator error  $K_{\Delta_i}$ , so we will introduce an estimator,  $\hat{K}_i$ , that utilizes known quantities. We propose the following online estimator for the robots to use

$$\begin{aligned} \dot{\hat{K}}_i &= \lambda_i - \hat{K}_i \Lambda_i \\ \dot{\lambda}_i &= \dot{p}_i (C_{W_i} - p_i)^T \\ \dot{\Lambda}_i &= (C_{W_i} - p_i)(C_{W_i} - p_i)^T. \end{aligned} \quad (6)$$

Integrating  $\dot{\lambda}_i$  and  $\dot{\Lambda}_i$ , we can simplify this expression as

$$\begin{aligned} \hat{K}_i &= \tilde{K}_i \int_0^t (C_{W_i}(\tau) - p_i(\tau))(C_{W_i}(\tau) - p_i(\tau))^T d\tau \\ &= -\tilde{K}_i \Lambda_i(t) \end{aligned} \quad (7)$$

where  $\tilde{K}_i = (\hat{K}_i - K_i)$ . Note that while (6) and (7) are mathematically equivalent, the robots can only directly compute (6) because they do not have knowledge of the true error  $\tilde{K}_i$ . However, the form in (7) is useful for analysis.

#### B. Controller Performance

The behavior of our system with our positional control law, weightings adaptation law, and  $\hat{K}_i$  estimator is formalized in the following theorem.

*Theorem 1:* Using the positional control law (4), weightings adaptation law (5), and estimator for  $\hat{K}_i$  (6), the robots converge to the centroids of their weighted Voronoi cells,

$$\lim_{t \rightarrow \infty} \|p_i(t) - C_{W_i}(t)\| = 0 \quad \forall i \in \{1, \dots, n\}. \quad (8)$$

Furthermore, the control gain matrix estimation error converges to the null space of  $\Lambda_i(t)$ ,

$$\lim_{t \rightarrow \infty} \tilde{K}_i(t) \Lambda_i(t) = 0 \quad \forall i \in \{1, \dots, n\}. \quad (9)$$

*Proof:*

To prove (8) and (9), we will invoke a global version of LaSalle's Invariance Principle ([19], Theorem 1.20). We will first introduce a continuously differentiable Lyapunov-like function  $V$ . We use this to show that show all trajectories of the system are bounded, and that the function is non-increasing,  $\dot{V} \leq 0$ . We then use LaSalle's Principle to prove the claim of the theorem. Consider the function

$$\begin{aligned} \mathcal{V} &= \sum_{i=1}^n \int_{W_i} \frac{1}{2} (\|q - p_i\|^2 - w_i) \phi(q) dq \\ &\quad + \sum_{i=1}^n \frac{1}{2} \text{Tr}[\tilde{K}_i \tilde{K}_i^T] \end{aligned}$$

with derivative

$$\begin{aligned} \dot{\mathcal{V}} &= \sum_{i=1}^n \int_{W_i} (q - p_i)^T \phi(q) dq \dot{p}_i - \sum_{i=1}^n \int_{W_i} \frac{1}{2} \phi(q) dq \dot{w}_i \\ &\quad + \sum_{i=1}^n \text{Tr}[\dot{\hat{K}}_i \tilde{K}_i^T]. \end{aligned}$$

We can break this into three parts as

$$\begin{aligned} \dot{\mathcal{V}}_1 &= \sum_{i=1}^n \int_{W_i} (q - p_i)^T \phi(q) dq \dot{p}_i, \quad \dot{\mathcal{V}}_2 = \sum_{i=1}^n \frac{-1}{2} M_{W_i} \dot{w}_i. \\ \dot{\mathcal{V}}_3 &= \sum_{i=1}^n \text{Tr}[\dot{\hat{K}}_i \tilde{K}_i^T]. \end{aligned}$$

By plugging in our controller (4) for  $\dot{p}_i$ , the time derivative  $\dot{\mathcal{V}}_1$  becomes

$$\begin{aligned} \dot{\mathcal{V}}_1 &= \sum_{i=1}^n \int_{W_i} (q - p_i)^T \phi(q) dq [K_i (C_{W_i} - p_i)] \\ &= \sum_{i=1}^n -M_{W_i} (C_{W_i} - p_i)^T K_i (C_{W_i} - p_i). \end{aligned}$$

Given that  $K_i$  is positive definite and  $M_{W_i}$  is positive, we know that overall  $\dot{V}_1 \leq 0$ .

For  $\dot{V}_2$ , plugging in our controller (5) for  $\dot{w}_i$  yields

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^n \frac{k_w}{2} \sum_{j \in \mathcal{N}_i} \left( (w_i - f(\hat{K}_i)) - (w_j - f(\hat{K}_j)) \right) \\ &= 0. \end{aligned}$$

For  $\dot{V}_3$ , we see that

$$\dot{V}_3 = \text{Tr} \left[ -\tilde{K}_i \Lambda_i(t) \tilde{K}_i^T \right] \leq 0.$$

Overall,  $\dot{V} \leq 0$ . Given this, we see that the trajectories of both  $p_i(t)$  and  $\tilde{K}_i(t)$  are bounded, which also implies that  $\hat{K}_i$  is bounded.

To determine whether the weightings are bounded, we can consider the vector form  $\dot{w}$  as

$$\dot{w} = -k_w M^{-1} L w + k_w M^{-1} L f(t) \quad (10)$$

where  $M^{-1}$  is a diagonal matrix of positive entries,  $L$  is the Laplacian of the graph in which all Voronoi neighbors share an edge (the Delaunay graph), and

$$f(t) = \begin{bmatrix} f(\hat{K}_1(t)) \\ \dots \\ f(\hat{K}_n(t)) \end{bmatrix}.$$

It is known that the graph Laplacian  $L$  is positive semi-definite ([20], [21]), and it can be shown that the product  $M^{-1}L$  is positive semi-definite. Therefore  $w(t)$  is the state of a marginally stable filter defined by (10). Also, since  $\hat{K}_i$  is bounded for all  $i$ ,  $f(t)$  is bounded, and we have that the input to the filter defining  $w(t)$  is bounded. From the input-to-state properties of marginally stable filters we know that for  $w(t)$  to go unbounded the driving signal  $k_w M^{-1} L f(t)$  must lie in the null space of the dynamics matrix  $-k_w M^{-1} L$  of the filter. However, since the input  $f(t)$  is itself multiplied by  $k_w M^{-1} L$ , the driving signal  $k_w M^{-1} L f(t)$  can have no component in the null space of  $-k_w M^{-1} L$ , hence  $w(t)$  remains bounded. This shows that all trajectories of the system ( $p_i(t)$ ,  $\tilde{K}_i(t)$ , and  $w_i(t)$  for all  $i$ ) remain bounded.

Notice that we have already showed that  $\dot{V} \leq 0$ . Therefore, to complete the proof, we have to find the largest invariant set within the set defined by  $\dot{V} = 0$ . We can see that  $\dot{V} = 0$  occurs when  $p_i = C_{W_i}$ , and  $\tilde{K}_i \Lambda_i = 0$ . From our control law (4) and estimator (7), we can see that this is itself an invariant set. Therefore, by LaSalle's Invariance Principle, we can say that the positions of the robots obey

$$p_i(t) \rightarrow C_{W_i}(t) \text{ as } t \rightarrow \infty$$

and

$$\tilde{K}_i(t) \Lambda_i(t) \rightarrow 0 \text{ as } t \rightarrow \infty,$$

proving (8) and (9) from Theorem 1.  $\blacksquare$

*Corollary 1:* If  $\Lambda_i(t)$  achieves full rank for all  $i \in \{1, \dots, n\}$  and any  $t > 0$ , then

$$\lim_{t \rightarrow \infty} \hat{K}_i(t) = K_i \quad \forall i \in \{1, \dots, n\}. \quad (11)$$

Furthermore,

$$\lim_{t \rightarrow \infty} (w_i(t) - w_j(t)) = f(K_i) - f(K_j) \quad (12)$$

for all  $i, j \in \{1, \dots, n\}$ .

*Proof:* In Theorem 1, we stated that  $\tilde{K}_i(t)$  converges to the null space of  $\Lambda_i(t)$ . It can be shown that the rank of  $\Lambda_i(t)$  is nondecreasing in time. Hence if at some  $\tau > 0$ ,  $\Lambda_i(\tau)$  has full rank, then  $\Lambda_i(t)$  has full rank for all  $t \geq \tau$ , and the null space of  $\Lambda_i(t)$  is the set comprised of the zero vector. Therefore for  $\tilde{K}_i = (\hat{K}_i - K_i)$ ,

$$\begin{aligned} \lim_{t \rightarrow \infty} \tilde{K}_i \Lambda_i &= 0 \\ \Rightarrow \lim_{t \rightarrow \infty} \hat{K}_i &= K_i, \end{aligned}$$

proving (11). Furthermore, as shown before, our weightings adaptation law (5) can be written in vector form as (10). By the properties of stable linear filters, we know that as the input approaches a limit, the state will also approach a limit [22], which will satisfy the steady state equation

$$w \rightarrow \{w_\infty | 0 = -k_w M^{-1} L (f_\infty - w_\infty)\}$$

where  $f_\infty$  is the limit of  $f(t)$ , written as

$$f_\infty = \begin{bmatrix} f(K_1) \\ \vdots \\ f(K_n) \end{bmatrix}.$$

Solving for  $w_\infty$  we find

$$\begin{aligned} k_w M^{-1} L w_\infty &= k_w M^{-1} L f_\infty \\ L w_\infty &= L f_\infty. \end{aligned}$$

Given that  $L$  is the Graph Laplacian, this is equivalent to

$$w_{i,\infty} - w_{j,\infty} = f(K_i) - f(K_j),$$

proving (12) from Corollary 1.  $\blacksquare$

*Remark 1:* In practice, we have seen in every simulation and experiment that  $\Lambda_i(t)$  quickly achieves full rank for all  $i$ . To see why, notice that for  $\Lambda_i(t)$  not to achieve full rank, the robot  $i$  must move in a precisely straight line throughout its entire trajectory, which is highly unlikely given the nonlinear nature of the system. However, this fact is difficult to prove rigorously.

#### IV. SIMULATIONS

To demonstrate our move-to-centroid controller (4), weightings adaptation law (5), and  $\hat{K}_i$  estimator (6), we conducted simulations in Matlab. Using a rectangular environment  $Q$  with a constant density function  $\phi(q)$ , the results demonstrate the performance of our controller, adaptation law, and estimator. Here, we chose the following function to measure the actuation performance

$$f(\hat{K}_i) = \|\hat{K}_i\|.$$

All the agents have a zero  $K_{\Delta_i}$  matrix, except agent 4, which has  $K_{\Delta_4} = [-0.4, 0.0; 0.0, -0.4]$ . The weights are initially set to one, except agent 2, which is assigned a lower weight

$w_2 = 0.5$ . Figure 1 shows the initial and final positions of the agents and Figure 2 shows the cost function and the performance of the agents,  $w_i - \|\hat{K}_i\|$ . The weight of agent 4 is in red, and the weight of agent 2 is in green.

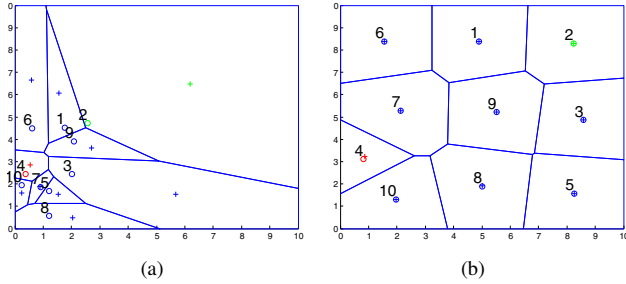


Fig. 1. (a) Initial and (b) final configurations of the robots.

In the final configuration, we see that robot 4 has a smaller area relative to its neighbors, due to the decreased performance weight. The cost also decreased over time, shown in Figure 2, settling to a local minimum. As expected,  $(w_i - \|\hat{K}_i\|)$  converges to a common value across all agents.

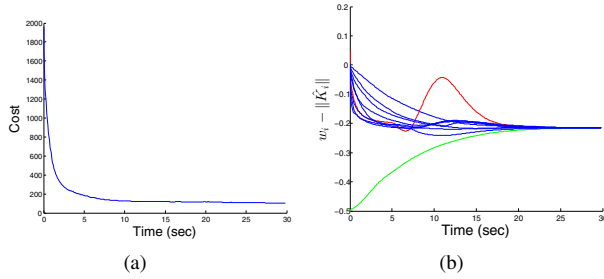


Fig. 2. (a) Cost and (b)  $(w_i - \|\hat{K}_i\|)$ .

Finally, Figure 3 illustrates the convergence of the estimator  $\hat{K}_i$  to  $K_i$  over time. Note that we only expect the estimator of agent 4 to change, shown in red. Over the course of the simulation, the  $\hat{K}_4$  converges to the true value,  $K_4$ .

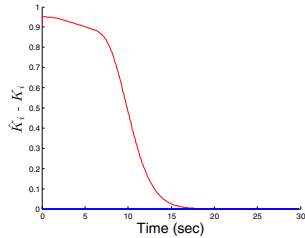


Fig. 3. Convergence of  $\|\hat{K}_i\|$  to  $\|K_i\|$

## V. EXPERIMENTS

In order to verify the behavior of our controller, we implemented our algorithm using m3pi<sup>1</sup> robots equipped

<sup>1</sup>Pololu’s m3pi: [www.pololu.com/product/2151](http://www.pololu.com/product/2151)

with XBee<sup>2</sup> radios. The m3pi robot utilizes an onboard mbed microcontroller to handle actuation and communication. The mbed handles the control of the motors on the robot, and we send velocity data via XBee radios to the robots based on the Voronoi calculations in Matlab. To localize our robots, we used NaturalPoint’s OptiTrack<sup>3</sup> system. Each robot was equipped with a unique configuration of IR markers, tracked by the OptiTrack system. A video with the experimental runs is attached to accompany this paper and can be found on the Multi-Robot Systems Lab website. For our weightings adaptation law, we use the performance function

$$f(\hat{K}_i) = \|\hat{K}_i\|.$$

We also incorporated a low-level point-offset controller [23] to account for the nonholonomic dynamics of the m3pis. Short-throw projectors were used to display the centroid and Voronoi boundaries during the experiments.

In this experiment, our rectangular environment  $Q$  had a constant density function  $\phi(q)$ . All six robots were assigned initial weights of  $w_i = 1$ , except for robot 2 and robot 6 with  $w_2 = 0.7$  and  $w_6 = 1.1$ . Robots were given a zero  $K_{\Delta_i}$  matrix, except robot 1, which was assigned  $K_{\Delta_1} = [-0.3, 0; 0, -0.3]$ . Figure 4 shows the initial and final configurations of the agents.

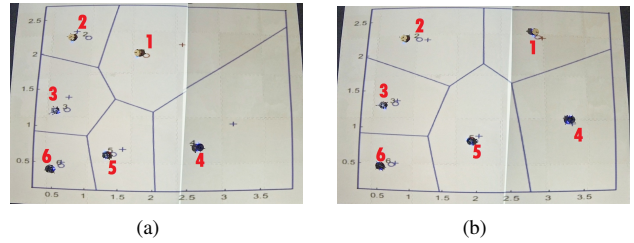


Fig. 4. (a) Initial and (b) final configurations of the robots.

In Figure 5(a) we can see that the cost function decreases over time to some minimum value, implying that the group converged to a locally optimal configuration. We can also see the successful convergence of our estimator  $\hat{K}_i$  to  $K_i$  in Figure 5(b).

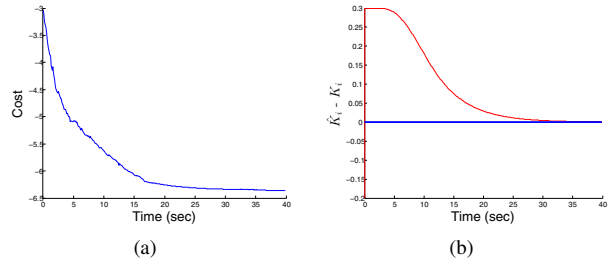


Fig. 5. (a) Cost and (b)  $\hat{K}_i$  estimator.

Figure 6(a) shows the weightings over time. We see that agent 1 has the lowest weight over time, which corresponds

<sup>2</sup>Digi’s XBee: [www.digi.com/xbee/](http://www.digi.com/xbee/)

<sup>3</sup>Natural Point OptiTrack: [www.naturalpoint.com/optitrack/](http://www.naturalpoint.com/optitrack/)

to its lower relative performance. Figure 6(b) shows that the difference  $(w_i - \|\hat{K}_i\|)$  converges to a common value.

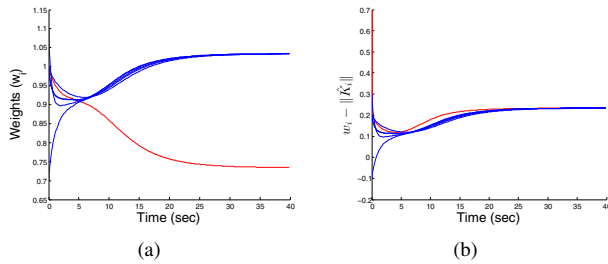


Fig. 6. (a) Performance weightings and (b)  $(w_i - \|\hat{K}_i\|)$ .

## VI. CONCLUSION

In this paper, the authors have built upon their previous method of using adaptive weightings to adjust for individual variations in performance within multi-robot coverage control. To account for errors in actuation, the robots compare values of an error estimate with their neighbors, and using an adaptive weightings law, change the value of their weightings. By controlling these weights, we are able to modify the Voronoi boundaries between neighboring robots, which adjusts a robot's cell size relative to its neighbors. The weightings adaptation law and error estimation occur online within the coverage control algorithm. The positional controller is similar to previous implementations of Voronoi coverage control. We illustrate the success of our algorithm using m3pi robots.

Our method incorporates actuation error into the decentralized algorithm while maintaining stability and performance. This can provide an additional level of robustness in real-world applications when the robots are in an unknown environment. This can be achieved by adjusting for internal variations in actuation, as well as robustness against external factors such as rough terrain. It can also provide insight into identifying failures of a robot's actuation. A current limitation of the algorithm is the conservative assumption that for each robot,  $K_i > 0$ . Future work may relax this restraint. Other extensions may study the robustness of this algorithm to malicious agents. Another direction could tie to the author's previous work, where the weightings quantify both sensing and actuation performance within the group.

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